Shaft CenterLINES

Margin of Stability

What it is, how to measure it...

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NOTATION

D,K,M . . Rotor modal damping, stiffness and mass, respectively

λ . . . . . Fluid circumferential average velocity ratio

Ω . . . . . Rotative speed

ω . . . . . Exciting force speed

$j=\sqrt{-1}$

A fundamental concern when purchasing, or specifying the performance parameters of, rotating machinery with fluid bearings is, "Will my machine be stable?," or alternatively, " When will my machine be UNstable?" The purpose of this article is to provide some information about stability, specifically, what stability is, how to measure it, and what factors contribute to stability or the lack thereof. In the last issue of *Orbit*, we presented a discussion of the synchronous amplification factor, Q_S. This article is an extension of that previous one, since amplification factor and stability are frequently mentioned in the same breath. It is important to note that the amplification factor is NOT a measure of stability.

Stability is defined as the resistance of a system to disturbance. For the purposes of this article, stability is concerned with the rotor's resistance to lateral motion, given the presence of some disturbing force on the rotor.

Stability can also be defined in terms of its negative, instability. The condition of instability dictates that the amplitude of the rotor's response motion will increase without bound even without any external disturbing force. In reality, it is impossible for motion to increase without bound. Typically, the rotor motion will increase until system nonlinearities (for instance, journal contact with the bearing surface) restrain the rotor in some sort of limit cycle of self-excited vibration.

But within the linear range of the system, the forced response motion of the rotor model can be summarized as:

$$ROTOR\ RESPONSE\ MOTION = \frac{FORCE}{STIFFNESS}$$

where, for a simply supported rotor with one lateral mode, the Dynamic Stiffness is defined as follows [1]:

Total Dynamic Stiffness =
Direct Dynamic Stiffness +
Quadrature Dynamic Stiffness

$$K - M\omega^2 \equiv \textit{Direct Dynamic Stiffness}\ (DDS) \qquad (1)$$

$$jD(\omega-\lambda\Omega) \equiv \textit{Quadrature Dynamic Stiffness}\ (QDS) \qquad (2)$$

There is a resonance when either of these terms become zero. When the *DDS* becomes zero, it is mechanical resonance by definition. The only term limiting the amplitude of motion at mechanical resonance is the Quadrature Dynamic Stiffness. This observed resonance peak has generally been called the "critical." When *QDS* becomes zero, it is fluid induced resonance. The only term limiting the peak of the motion at fluid induced resonance is the Direct Stiffness term. For the typical case of a lightly damped machine, the mechanical resonance will be the dominant factor determining system response.

In a simple spring-mass-damper system, the only Quadrature Stiffness term is $D\omega$ (damping times excitation frequency). This is why many books, past and present, use the term "damping becomes negative" or "damping becomes zero." In fact, it is impossible for damping *(D)* to be negative. It is the Quadrature Dynamic Stiffness that becomes negative [2]. Once more: When you see a book or paper that states that the damping goes negative, beware.

Both resonances are always present, regardless of machine parameters, however, the fluid-induced resonance has been rarely observed, because it requires nonsynchronous excitation. To illustrate these resonances isolated from each other, Figure 1(a) shows a Bode and polar plot of system response to nonsynchronous perturbation (exciting force proportional to ω^2, as with unbalance) with *QDS* artificially held to a constant, nonzero value (*QDS* must be nonzero or the amplitude at resonance is infinite). Similarly, Figure 1(b) shows a Bode and polar plot of system response to nonsynchronous perturbation with *DDS* artificially held to a constant nonzero value. The combined system response, including both *DDS* and *QDS* influence (not constant), is shown in Figure 1(c). The machine parameters are the same for all cases (Fig, 1a, b, and c). Please note that these are artificial conditions created to illustrate basic behavior.

From Eq.(1), the mechanical resonance frequency $\omega_{res(DDS)}$, occurs when $DDS = 0$, thus

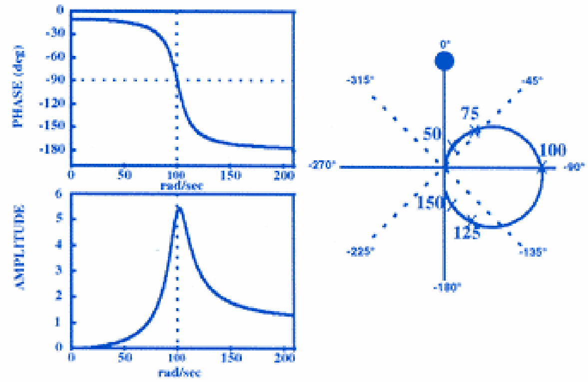
$$\omega = \omega_{res(DDS)} \equiv \sqrt{\frac{K}{M}} \qquad (3)$$

Similarly, from Eq. (2), the fluid-induced resonance occurs at an excitation speed that causes *QDS* to become equal to zero:

$$\omega = \omega_{res(QDS)} \equiv \lambda\Omega \qquad (4)$$

An eigenvalue analysis of the rotor model [1] will show that the rotor with fluid bearings is unstable if the mechanical resonance occurs at a lower speed than the fluid-induced resonance. This information, combined with Eqs. (3) and (4), allows us to derive an expression for the Margin of Stability. The Margin of Stability is defined as the perturbation speed difference between zeros (roots) of Direct and Quadrature Dynamic Stiffness as functions of frequency. The margin of stability has units of frequency (rad/sec).

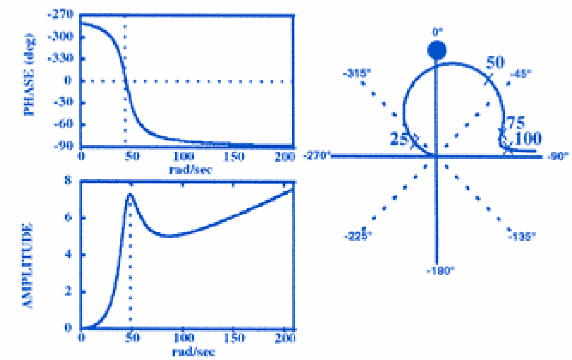
(a)



Mechanical resonance; shaft speed = 100 rad/sec.

$$K - M\omega^2 + j\,1872.5$$

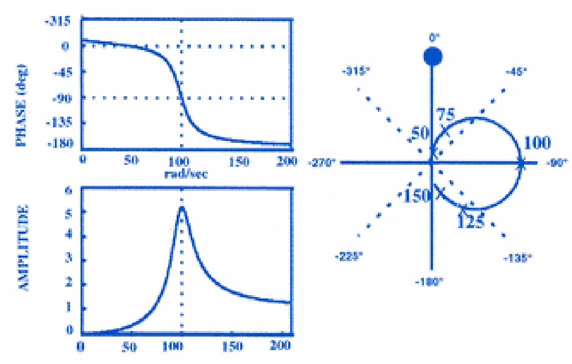
(b)



Fluid induced resonance; shaft speed = 100 rad/sec.

$$297.75 + jD(\omega - \lambda\Omega)$$

(c)



System resonance; shaft speed = 100 rad/sec.

$$K - M\omega^2 + jD(\omega - \lambda\Omega)\ 1872.5$$

Parameters

K = le4 M = 1 D = 35 λ = .45 Ω = *100* rad/sec

Figure 1

Bode and polar plots of isotropic rotor one lateral mode response to nonsynchronous excitation

$$(MS) = \omega_{res(DDS)} - \omega_{res(QDS)}$$

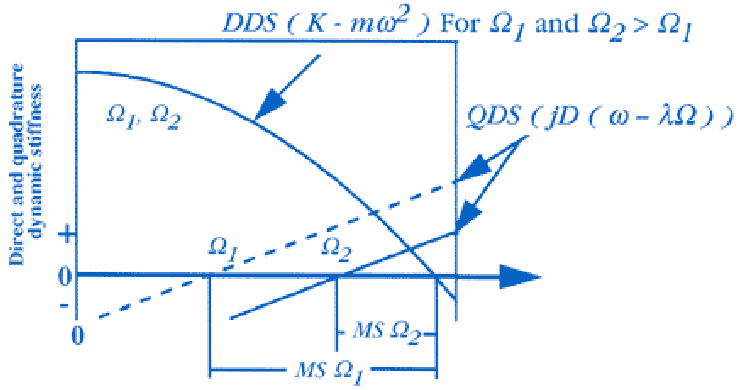
$$\equiv \sqrt{\frac{K}{M}} - \lambda\Omega \qquad (5)$$

pretation of the Margin of Stability with emphasis on the effect of changing shaft speed, Ω. As Ω increases, the zero crossing of *QDS* moves closer to the zero crossing of *DDS*, thus reducing the Margin of Stability.

In order to be able to compare various systems, it is reasonable to introduce the Nondimensional Margin of Stability (*NMS*) as follows:

$$(NMS) = \frac{(MS)}{\omega_{res(DDS)}} \equiv \frac{\sqrt{\frac{K}{M}} - \lambda\Omega}{\sqrt{\frac{K}{M}}} \qquad (6)$$

At the numerator of Eq. (6) there is the Margin of Stability; the denominator contains the direct resonance frequency. For stable systems, the value may vary between one and zero. A negative value means that the system is unstable. The Nondimensional Margin of Stability better describes and quantifies the system stability situation than the Margin of Stability.



Plot of Direct and Quadrature Dynamic Stiffness illustrating change in Margin of Stability (MS) as rotor speed (Ω) increases. Dashed line corresponds to Ω_1, with Ω_1 less than Ω_2. Note that *DDS* dos not change. *DDS* is independent of rotor speed. The zeros of *QDS* are $\omega = \lambda\Omega_1$ and $\omega = \lambda\Omega_1$ respectively.

The zero of *DDS* occurs at: $\omega = \sqrt{K / M}$

Figure 2

Conclusion

We have discussed the Margin of Stability and Nondimensional Margin of Stability as a means of evaluating stability. It is assumed that the machine parameters (K, M, D, λ) are known. In this case, practical application of the formulas for stability is as easy as plugging these values into the aforementioned equations. There need be no guesswork when evaluating stability of rotor systems using these methods.

These parameters are not commonly known, however, they are readily obtainable. In fact, if data has been collected and stored (as from balancing) it may be possible to find these parameters from existing data without disturbing the machine. If purchasing a new machine, it is possible for the vendor to supply data from which parameters can be calculated. References [3] and [4] describe in detail how to go about finding the machine parameters, and are available by contacting the *Orbit* magazine.

The pioneering work of defining the rotor modeling techniques that incorporate these parameters, along with the techniques for experimentally determining them, has been progressing for many years. The results from the work of engineers, such as J.M. Stone and A.F. Underwood at General Motors in 1946, E.H. Hull at General Electric in 1959, and, more recently, by Donald Bently and Bently Rotor Dynamics Research Corporation, have led to accurate modeling techniques that have proven invaluable tools for machinery diagnostics and design.

This article is part of a series that presents discussions of the fundamentals of quantifying and discussing rotor response, in the interest of promoting more consistent and correct communication between technical professionals concerned with rotating machinery. In the next issue of the *Orbit,* we extend the results of this and the previous article, *Synchronous Amplification Factor, Qs* [5], by presenting the more general case of nonsynchronous excitation. ■

References:

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